

SWL-18

Mathematics

(INTERMEDIATE PART - I, Class 11th) - (IV)

Paper : I

Time: 30 Minutes

Objective Code : 6197

Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. i. $\frac{n!}{(n-r)!}$ is always equal to

- (A) ${}^n P_r$ (B) ${}^n C_r$ (C) ${}^r P_n$ (D) ${}^r C_n$

ii. If $1/a$, $1/b$ and $1/c$ are in G.P then common ratio is equal to:

- (A) $\pm\sqrt{\frac{c}{a}}$ (B) $\pm\sqrt{\frac{a}{c}}$ (C) $\pm\sqrt{a+c}$ (D) $\pm\sqrt{a-c}$

iii. Sum of n - arithmetic means between a and b is equal to :

- (A) $\frac{a-b}{2}$ (B) $n(\frac{a-b}{2})$ (C) $\frac{a+b}{2}$ (D) $n(\frac{a+b}{2})$

iv. $\frac{A}{x-1} + \frac{B}{x+1}$ is a partial fraction form of the proper fraction:

- (A) $\frac{1}{x^2-1}$ (B) $\frac{1}{x^3-1}$ (C) $\frac{1}{x^2+1}$ (D) $\frac{1}{x^3+1}$

v. If 'x-2' is a factor of polynomial $x^3 + 2x^2 + kx + 4$ then k equals :

- (A) 10 (B) -10 (C) 2 (D) 4

vi. The sum of all cube roots of unity equals:

- (A) 1 (B) ω (C) 0 (D) ω^2

vii. Let $A = \begin{bmatrix} 1 & 2 & 3x \\ 2 & 3 & 6x \\ 3 & 5 & 9x \end{bmatrix}$ then $|A|$ is equal to:

- (A) 1 (B) 3 (C) 2 (D) 0

viii. If A is a square matrix and $A' = A$, then A is called

- (A) hermitian matrix (B) skew hermitian matrix (C) symmetric matrix (D) skew symmetric matrix

ix. If 'p' is a logical statement, then $p \wedge \sim p$ is always:

- (A) absurdity (B) contingency (C) tautology (D) conditional

x. If $(x+iy)^2 = a+ib$ then $x^2 - y^2$ equals:

- (A) $a^2 + b^2$ (B) $a^2 - b^2$ (C) $a-b$ (D) $a+b$

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xi. Period of $\cot x/2$ is equal to :

- (A) 2π (B) 4π (C) π (D) 3π

A coin is tossed twice then probability of getting all heads equal:

- xii. (A) $1/2$ (B) $1/3$ (C) $1/4$ (D) $2/3$

xiii. $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$ is called

- (A) Sine law (B) Cosine Law (C) Tangent law (D) Fundamental law

xiv. If α , β and γ are angles of triangle ABC, then $\cos\left(\frac{\alpha+\beta}{2}\right)$ will be equal to :

- (A) $\sin \alpha$ (B) $\sin \gamma$ (C) $\sin \frac{\gamma}{2}$ (D) $\sin \beta$

xv. In an oblique triangle ABC, if $a=2$ and $\alpha=30^\circ$, then circum-radius 'R' is equal to:

- (A) 4 (B) 3 (C) 1 (D) 2

xvi. $\sin^{-1}\left[A\sqrt{1-B^2} + B\sqrt{1-A^2}\right]$ is equal to

- (A) $\cos^{-1}A + \cos^{-1}B$ (B) $\cos^{-1}A - \cos^{-1}B$ (C) $\sin^{-1}A + \sin^{-1}B$ (D) $\sin^{-1}A - \sin^{-1}B$

xvii. The solution of $\sin x = -\frac{\sqrt{3}}{2}$ in interval $[0, 2\pi]$ equals

- (A) $\frac{4\pi}{3}, \frac{2\pi}{3}$ (B) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (C) $\frac{\pi}{3}, \frac{5\pi}{3}$ (D) $\frac{4\pi}{3}, \frac{\pi}{3}$

xviii. Second term in the expansion of $(\sqrt{x} + \frac{1}{2x^2})^{10}$ equals:

- (A) $5x^{2/5}$ (B) $10x^{2/5}$ (C) $10x^{5/2}$ (D) $5x^{5/2}$

xix. If the number of terms in the expansion of $(a+b)^n$ is 16 then 'n' equals

- (A) 18 (B) 16 (C) 17 (D) 15

xx. Length 'l' of an arc of a circle with radius r and central angle θ is equal to :

- (A) $r^2\theta$ (B) $r\theta$ (C) $r\theta^2$ (D) $\frac{1}{2}r^2\theta$

Mathematics

Time : 2:30 Hours

(SUBJECTIVE)

Note : Section I is compulsory. Attempt any three (3) questions from section II.

(SECTION - I)

2. Write short answers to any Eight parts :

(8 × 2 = 16)

- i. Prove that $\frac{-7}{12} - \frac{5}{18} = \frac{-21-10}{36}$.
- ii. Separate $4+5i$ in to real and imaginary parts.
- iii. If $\forall z_1, z_2 \in C$ prove that $z_1 + z_2 = z_2 + z_1$.
- iv. Show A-B and B-A by Venn diagram when A and B are overlapping sets. (1+1=2)
- v. Show that the statement $\sim (p \rightarrow q) \rightarrow p$ is a tautology.
- vi. Find the inverse of the relation $\{(x, y) / y = 2x + 3, x \in \mathbb{R}\}$.
- vii. Solve the matrix equation $3X - 2A = B$. If $A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$
- viii. If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the values of a and b.
- ix. Define co-factor of an element of a matrix.
- x. Solve by completing square $x^2 - 3x - 648 = 0$.
- xi. Solve the equation : $x^{2/5} + 8 = 6x^{1/5}$.
- xii. Show that $x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$ where ω is the cube root of unity.

3. Write short answers to any Eight parts :

(8 × 2 = 16)

- i. Resolve $\frac{x^2+1}{x^2-1}$ into partial fraction. (1+1=2)
- ii. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P, show that common ratio is $\pm \sqrt{\frac{a}{c}}$. (1+1=2)
- iii. Which term of A.P 5, 2, -1, is -85? (1+1=2)
- iv. Define G.M between two numbers 'a' and 'b' and show that (G.M) geometric mean $= \pm \sqrt{ab}$ (1+1=2)
- v. In series $y = 1 + 2x + 4x^2 + 8x^3 + \dots$, show that $x = \frac{y-2}{2y}$. (1+1=2)
- vi. If $a = -2, b = -8$, find G and H, also show that $G < H$ for $(G < O)$ with usual notation. (1+1=2)
- vii. Prove that ${}^n P_r = n \cdot {}^{n-1} P_{r-1}$, where p is permutation.
- viii. How many necklaces can be made from 6 beads of different colours? (1+1=2)
- ix. How many triangles can be formed by joining the vertices of polygon having 8 sides? (1+1=2)
- x. Show that $1 + 2 + 4 + \dots + 2^{n-1} = 2^n - 1$ is true for $n = 4, 5$. (1+1=2)
- xi. Show that $n^2 + n$ is divisible by 2 for $n = 2, 3$. (1+1=2)
- xii. Expand $(2-3x)^{-1}$ up to three terms. (1+1=2)

4. Write short answers to any Nine parts :

(9 × 2 = 18)

- i. Evaluate $\frac{\tan \pi/3 - \tan \pi/6}{1 + \tan \pi/3 \tan \pi/6}$.
- ii. Prove that $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{\cot \theta - 1}{\cot \theta + 1}$.

iii. Prove that $(\cos\theta - \tan\theta)^2 = \frac{(1 - \sin\theta)}{1 + \sin\theta}$.

iv. By using fundamental law of trigonometry, show that $\sin(\pi/2 + \alpha) = \cos\alpha$.

v. Prove that $\sin(45^\circ + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$.

vi. Find the period of $\cot 8x$.

vii. Find that value of $\cos 2\alpha$ for $\cos\alpha = 3/5$ where $0 < \alpha < \pi/2$.

viii. If $\beta = 60^\circ$, $\gamma = 15^\circ$, $b = \sqrt{6}$ then find c and α for any triangle ABC.

ix. Find the area of triangle, given two sides and their included angle,
 $a = 4.33$, $b = 9.25$, $r = 56^\circ 44'$.

x. Show that $r_1 = s \tan \frac{\alpha}{2}$.

xi. Find the value of the expression $\operatorname{Cosec}(\tan^{-1}(-1))$.

xii. Find the solutions of $\sin x = -\sqrt{\frac{3}{2}}$ in $[0, 2\pi]$.

xiii. Find the value of θ , satisfying the equation $3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0$.

(SECTION - II)

Note: Attempt any three (3) questions:

(3 X 10 = 30)

5. (a) Convert $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ to logical form and prove by constructing truth table. (2 + 3)

(b) Use Cramer's rule to solve the system:

$$\left. \begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 2x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned} \right\}$$

(2+1+1+1)

6. (a) Prove that $\frac{x^2}{a^2} + \frac{(mn+c)^2}{b^2} = 1$ will have equal roots, if $c^2 = a^2m^2 + b^2$ (5)

(b) Resolve into partial fraction:

$$\frac{x^3 + 2x + 2}{(x^2 + x + 1)^2}$$

(5)

7. (a) Find the sum of an infinite series $r + (1+k)y^2 + (1+k+k^2)r^3 + \dots$ (5)

(b) Find the co-efficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^2}$. (5)

8. (a) Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$ (5)

(b) Reduce $\sin^4\theta$ to an expression involving only function of multiples of θ , raised to the first power. (5)

9. (a) Prove that: $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$ using usual notations. (5)

(b) Prove that $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{13} = \cos^{-1}\frac{253}{325}$. (5)